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Theory of conductivity in a quasi-two-dimensional system based on operator algebra technique

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Abstract
We describe a method of obtaining a conductivity formula which incorporates the relaxation functions in a quasi-two-dimensional system. A parabolic confinement potential is adopted. We derive the relaxation function using projectors. We expand the propagators via the series expansion of the diagonal projectors and obtain useful relations enabling one to calculate the relaxation functions.

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1. Introduction

It is well known that the dynamical transition behaviour of a semiconductor is characterized by the relaxation mechanism of the system. The main purpose of this work is to investigate the effect of the induced current due to polarizability on the transition mechanism when an external field is applied on a semiconductor. Several theories of quantum statistical mechanical approaches have been reported so far [1–12]. Also there are many perturbative expansion methods such as projection techniques [3–7, 9–12]. Compared with many other perturbative expansion methods, the projection operator technique is useful for the explanation of quantum phenomena of a semiconductor.

In recent years, in conjunction with the growth of solid-state technology, the behaviour of electrons in solids has received a great deal of attention. For the behaviour of electrons, the study of transport phenomena based on the conductivity formalism is known to be one of the most popular fields for the investigation of the microscopic scattering mechanism. Among

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many approaches we are interested in the projection technique, which produces an elegant formalism using projectors [3–7, 9–12].

The study of low-dimensional electron systems is of great importance in semiconductor physics at the present time. Theoretical studies of cyclotronic transitions in quasi-two-dimensional quantum-well structures have been in active progress over the last few years. The quantum well is a system in which the electron motion is restricted in one direction, thus producing quantum confinement. This is realized in silicon-based MOS structures with quantum wells formed at the semiconductor boundary and in single GaAs/$Al_xGa_{1-x}As$ heterojunctions, where the quantum well is created in the GaAs layer at the heterojunction boundary. We are interested in the confinement of electrons by parabolic potentials. To our knowledge, although the theories reported so far have been based on a rigorous statistical quantum mechanical methodology, they are limited in the sense that many-body effects are mainly treated at the first quantized level.

Here, we introduce a method of deriving the conductivity formula which includes the relaxation functions in a quasi-two-dimensional system. We use an operator algebra technique based on the second quantization formalism for the expansion of the propagators which are included in the conductivity tensor. Using the usual projection technique, we expand the relaxation factors contained in the conductivity tensor and obtain Lorentz-like formulae for the conductivity. A parabolic confinement potential is adopted. We shall describe how to derive the relaxation function using projectors [10]. We expand propagators via a series expansion of the diagonal projectors. This method of expansion is conventional in many theories [6, 10]. In our two-dimensional system, the properties of projection operators are quite different from those of three-dimensional systems. We derive the useful relations of equation (3.14) for the calculation of the relaxation functions. Using these relations and systematically calculating the matrix elements, we obtain a conductivity formalism which incorporates the relaxation function.

2. Review of the conductivity formula

We suppose that a linearly polarized electric field, $\vec{E}(t) = E_0\hat{z}[\exp(\mathrm{i}\omega t)+\exp(-\mathrm{i}\omega t)]$, applied along the z-axis, gives the optical absorption power

$$P(\omega) = \frac{E_0^2}{2}\mathrm{Re}\,[\sigma_{zz}(\omega)]$$

where 'Re' denotes 'the real part of' and $\sigma_{zz}(\omega)$ is the optical conductivity tensor [7].

We consider a system of electrons interacting weakly with a background of phonons and confined in a parabolic well with a characteristic frequency of ω_0. We choose a parabolic potential because it is well analysed and easy to handle. Then, the induced conductivity is given by [10]

$$\sigma_{zz}(\bar{\omega}) = \sum_\alpha \left(\frac{\mathrm{i}}{\bar{\omega}}\right) j_\alpha \langle Z(\bar{\omega})\rangle_\alpha + \sum_\alpha \left(\frac{\mathrm{i}}{\bar{\omega}}\right) j_\alpha^* \langle\!\langle Z(\bar{\omega})\rangle\!\rangle_\alpha \tag{2.1}$$

where

$$Z(\omega) = (\hbar\bar{\omega} - L)^{-1} J_z \tag{2.2}$$

and

$$\begin{aligned}\langle X\rangle_\alpha &\equiv T_R\{\rho[X, a_\alpha^+ a_{\alpha+1}]\}\\ \langle\!\langle X\rangle\!\rangle_\alpha &\equiv T_R\{\rho[X, a_{\alpha+1}^+ a_\alpha]\}.\end{aligned} \tag{2.3}$$

The system Hamiltonian is

$$H = H_e + H_p + V = \sum_{\alpha,\beta} \langle\alpha|h_e|\beta\rangle a_\alpha^+ a_\beta + \sum_q \hbar\omega_q b_q^+ b_q + \sum_q \sum_{\kappa,\lambda} v_{\kappa\lambda}(q) a_\kappa^+ a_\lambda (b_q + b_-q^+) \tag{2.4}$$

$$L = L_e + L_p + L_v. \tag{2.5}$$

Here $\bar{\omega} = \omega + \mathrm{i}\eta$ and L is the Liouville operator corresponding to the Hamiltonian H. ρ is the equilibrium density matrix of the system, T_R denotes the many-body trace, $|\alpha\rangle$ and $|\beta\rangle$ are the electron state vectors characterized by proper quantum numbers, $q \equiv (\vec{q}, s)$, $\vec{q}$ being the phonon wavevector and s the polarization index, $\hbar\omega_q$ is the phonon energy and $v(q)$ is the electron–phonon couping factor. a_β^+ (a_β) creates (annihilates) an electron in the state β and b_q^+ (b_q) creates (annihilates) a phonon in the state q. Now we consider a system of electrons confined in a parabolic quantum well with characteristic frequency ω_0 in the z-direction. The Hamiltonian of the single electron is chosen as

$$h_e = -\frac{\hbar^2\nabla^2}{2m} + \frac{1}{2}m\omega_0^2 z^2. \tag{2.6}$$

Then the energy eigenvalue E_α and the eigenstate $|\alpha, k\rangle$, α being the quantum number, are given by

$$E_\alpha = (\alpha + 1/2)\hbar\omega_0 + \frac{\hbar^2(k_x^2 + k_y^2)}{2m} \tag{2.7}$$

$$|\alpha, k\rangle = \Psi_{\alpha,k} = \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/2} (2^\alpha \alpha!)^{-1/2} \exp(\mathrm{i}k_x x + \mathrm{i}k_y y) \exp\left(-\frac{m\omega_0}{2\hbar} z^2\right) H_\alpha\left(\sqrt{\frac{m\omega_0}{\hbar}} z\right) \tag{2.8}$$

where $H_\alpha(x)$ are the Hermite polynomials. The current operator of this system is given by

$$J_z = \sum_\alpha j_\alpha a_{\alpha+1}^+ a_\alpha + \sum_\beta j_\alpha^* a_\alpha^+ a_{\alpha+1} \equiv J_{z1} + J_{z2} \tag{2.9}$$

where

$$j_\beta = -\mathrm{i}e\left(\frac{\hbar\omega_\alpha(\alpha + 1)}{2m}\right)^{1/2}. \tag{2.10}$$

3. The expansion of the propagator and the function of relaxation factors

In order to reform equation (2.1) we further introduce two sets of projectors by

$$P_\alpha X \equiv J_z \frac{\langle X\rangle_\alpha}{\langle J_z\rangle_\alpha} \qquad P'_\alpha = 1 - P_\alpha \tag{3.1}$$

and

$$Q_\alpha X \equiv J_z \frac{\langle\langle X\rangle\rangle_\alpha}{\langle\langle J_z\rangle\rangle_\alpha} \qquad Q'_\alpha = 1 - Q_\alpha. \tag{3.2}$$

Now using the operator algebra technique the factors given in equation (2.1) can be obtained as

$$\langle Z(\bar{\omega})\rangle_\alpha = \frac{\langle J_z\rangle_\alpha/\hbar\bar{\omega}}{1 - \langle G_1 L J_z\rangle_\alpha/\langle J_z\rangle_\alpha} \tag{3.3}$$

$$\langle\langle Z(\bar{\omega})\rangle\rangle_\alpha = \frac{\langle\langle J_z\rangle\rangle_\alpha/\hbar\bar{\omega}}{1 - \langle\langle G_1 L J_z\rangle\rangle_\alpha/\langle\langle J_z\rangle\rangle_\alpha} \tag{3.4}$$

where G_1 is a new propagator defined by

$$G_1 = (\hbar\bar{\omega} - L P'_\alpha)^{-1}. \tag{3.5}$$

The properties of these operators are quite different from those of the three-dimensional cases, namely

$$L_d J_z = \sum_\alpha [\hbar\omega_\alpha J_z - 2\hbar\omega_\alpha j_\alpha^* a_\alpha^+ a_{\alpha+1}] \tag{3.6}$$

and

$$P' L_d J_z \neq 0. \tag{3.7}$$

Note that equation (3.6) for three-dimensional cases is much simpler than that for two-dimensional cases, so we split the Liouville operators appearing in the denominators into the diagonal part L_d and the interaction part L_v. Then we have

$$\langle G_1 L J_z\rangle = \langle G_1 L_d J_z\rangle + \langle G_1 L_v J_z\rangle \tag{3.8a}$$

and

$$\langle\langle G_1 L J_z\rangle\rangle = \langle\langle G_1 L_d J_z\rangle\rangle + \langle\langle G_1 L_v J_z\rangle\rangle. \tag{3.8b}$$

We expand the propagator G_1 using the conventional expansion method as in quantum transport theories [6, 10], so that

$$G_1 = G_2 \sum_{l=0}^{\infty} ((L_v P'_\alpha) G_2)^l \tag{3.9}$$

where the propagator G_2 is defined by

$$G_2 = (\hbar\bar{\omega} - L_d P'_\alpha)^{-1}$$

which can be expanded as

$$G_2 = \sum_{r=0}^{\infty} \left(\frac{1}{\hbar\bar{\omega}}\right)^{r+1} (L_d P'_\alpha)^r.$$

If we assume that the coupling is extremely weak, and dominated by pair interaction between electrons and phonons, than we have

$$\langle L_d P' X\rangle_\alpha = 0 \tag{3.10}$$

and

$$\begin{aligned} \langle Y L_d J_{z1}\rangle_\alpha &= \delta E_\alpha \langle Y J_{z1}\rangle_\alpha \\ \langle Y L_d J_{z2}\rangle_\alpha &= -\delta E_\alpha \langle Y J_{z2}\rangle_\alpha \\ \langle Y P' L_d J_z\rangle_\alpha &= -2\delta E_\alpha \langle Y J_{z2}\rangle_\alpha \\ \langle Y P' L_d J_{z2}\rangle_\alpha &= \langle Y L_d J_{z2}\rangle_\alpha \end{aligned} \tag{3.11}$$

where X and Y are arbitrary operators. We note that the relation (3.10) simplifies the calculations of the relaxation function for this system. As far as the operation of $\langle\langle\cdots\rangle\rangle$ is concerned, we obtain similar relations. Using these relations, equations (3.10) and (3.11) and the relation

$$\langle M P_\alpha L_v D\rangle = \langle M J_z\rangle\langle L_v D\rangle/\langle J_z\rangle = 0$$

where M is an arbitrary operator and D is an arbitrary diagonal operator, and applying the projection operator systematically, we may derive useful relations as follows:

$$\langle G_1 L_d J_z\rangle_\alpha = \left(\frac{1}{\hbar\bar{\omega}}\right)\langle L_d J_z\rangle_\alpha - 2\sum_{r=0}^{\infty}\left(\frac{1}{\hbar\bar{\omega}}\right)^{r+1} \langle L_v G_r L_v L_d^r J_{z2}\rangle \delta E_\alpha \tag{3.12a}$$

and

$$\langle\langle G_1 L_d J_z \rangle\rangle_\alpha = \left(\frac{1}{\hbar\bar{\omega}}\right) \langle\langle L_d J_z \rangle\rangle_\alpha + 2 \sum_{r=0}^{\infty} \left(\frac{1}{\hbar\bar{\omega}}\right)^{r+1} \langle\langle L_v G_r L_v L_d^r J_{z1} \rangle\rangle \delta E_\alpha \tag{3.12b}$$

where a propagator G_r is defined by

$$G_r = (\hbar\bar{\omega} - L_d)^{-1} \tag{3.13}$$

which can be expanded as $G_r = \sum_{r=0}^{\infty} (1/\hbar\bar{\omega})^{(r+1)} (L_d)^r$. We can also write

$$\langle G_1 L_v J_z \rangle = \langle L_v G_r L_v J_z \rangle \tag{3.14a}$$

and

$$\langle\langle G_1 L_v J_z \rangle\rangle = \langle\langle L_v G_r L_v J_z \rangle\rangle. \tag{3.14b}$$

In order to calculate the matrix elements, we use the anticommutation relations of fermions (electrons) and commutation relations of bosons (phonons). Using the relation

$$\langle (b_l + b_{-l}^+)(b_q + b_{-q}^+) \rangle_p = \langle (b_q + b_{-q}^+)(b_l + b_{-l}^+) \rangle_p = \{N_q + (N_q + 1)\}\delta_{l,-q} \equiv \tilde{N} \tag{3.15}$$

where N_q is the phonon distribution, and assuming weak interactions, we may take

$$\langle [a_\nu^+ a_\kappa (b_l + b_{-l}^+), a_\mu^+ a_{\alpha+1} (b_q + b_{-q}^+)] \rangle_p = [a_\nu^+ a_\kappa, a_\mu^+ a_{\alpha+1}] \langle (b_l + b_{-l}^+)(b_q + b_{-q}^+) \rangle_p. \tag{3.16}$$

Now we shall evaluate $\langle G_1 L_d J_z \rangle_\alpha$, $\langle G_1 L_v J_z \rangle_\alpha$, $\langle\langle G_1 L_d J_z \rangle\rangle_\alpha$ and $\langle\langle G_1 L_v J_z \rangle\rangle_\alpha$ using the matrix elements

$$\begin{aligned}
\langle j_{z1} \rangle &= \sum_\alpha j_{\alpha+1} \delta f_\alpha \\
\langle\langle j_{z2} \rangle\rangle &= \sum_\alpha j_\alpha (-\delta f_\alpha) \\
\langle L_d j_{z1} \rangle &= \sum_\alpha j_{\alpha+1} \delta E_\alpha \delta f_\alpha \\
\langle\langle L_d j_{z2} \rangle\rangle &= \sum_\alpha j_\alpha (-\delta E_\alpha)(-\delta f_\alpha) \\
\langle j_{z2} \rangle = \langle L_d j_{z2} \rangle &= \langle\langle j_{z1} \rangle\rangle = \langle\langle L_d j_{z1} \rangle\rangle = 0 \\
\langle L_v G_r L_v L_d^m j_{z1} \rangle &= \sum_\alpha \sum_q \tilde{Y}_1^1(q) \delta E_\alpha^m \delta f_\alpha \tilde{N}_q \\
\langle L_v G_r L_v L_d^m j_{z2} \rangle &= \sum_\alpha \sum_q \tilde{Y}_2^1(q) (-\delta E_\alpha)^m \delta f_\alpha \tilde{N}_q \\
\langle\langle L_v G_r L_v L_d^m j_{z1} \rangle\rangle &= \sum_\alpha \sum_q \tilde{Y}_3^1(q) \delta E_\alpha^m (-\delta f_\alpha) \tilde{N}_q \\
\langle\langle L_v G_r L_v L_d^m j_{z2} \rangle\rangle &= \sum_\alpha \sum_q \tilde{Y}_4^1(q) (-\delta E_\alpha)^m (-\delta f_\alpha) \tilde{N}_q.
\end{aligned} \tag{3.17}$$

Here we use the following abbreviations:

$$\begin{aligned}
\tilde{Y}_1^1(q) \equiv \sum_\mu & j_\alpha v_{\alpha+1,\mu}(l) v_{\mu,\alpha+1}(q) (\hbar\bar{\omega} - \epsilon_\mu + \epsilon_\alpha - \hbar\omega_q)^{-1} \\
& - j_\alpha v_{\alpha,\alpha}(l) v_{\alpha+1,\alpha+1}(q) (\hbar\bar{\omega} - \epsilon_{\alpha+1} + \epsilon_\alpha + \hbar\omega_q)^{-1} \\
& - j_\alpha v_{\alpha+1,\alpha+1}(l) v_{\alpha,\alpha}(q) (\hbar\bar{\omega} - \epsilon_{\alpha+1} + \epsilon_\alpha - \hbar\omega_q)^{-1} \\
& + \sum_\mu j_\alpha v_{\mu,\alpha}(l) v_{\alpha,\mu}(q) (\hbar\bar{\omega} - \epsilon_{\alpha+1} + \epsilon_\mu + \hbar\omega_q)^{-1}
\end{aligned} \tag{3.18}$$

$$\tilde{Y}_2^1(q) \equiv -j_\alpha^* v_{\alpha+1,\alpha}(l) v_{\alpha+1,\alpha}(q) (\hbar\bar{\omega} + \hbar\omega_q)^{-1} - j_\alpha^* v_{\alpha+1,\alpha}(l) v_{\alpha+1,\alpha}(q) (\hbar\bar{\omega} - \hbar\omega_q)^{-1} \tag{3.19}$$

$$\tilde{Y}_3^1(q) \equiv -j_\alpha v_{\alpha,\alpha+1}(l) v_{\alpha,\alpha+1}(q) (\hbar\bar{\omega} + \hbar\omega_q)^{-1} - j_\alpha v_{\alpha+1,\alpha}(l) v_{\alpha,\alpha+1}(q) (\hbar\bar{\omega} - \hbar\omega_q)^{-1} \tag{3.20}$$

$$\tilde{Y}_4^1(q) \equiv \sum_\mu j_\alpha^* v_{\alpha,\mu}(l) v_{\mu,\alpha}(q)(\hbar\bar{\omega} - \epsilon_\mu + \epsilon_{\alpha+1} - \hbar\omega_q)^{-1}$$
$$- j_\alpha^* v_{\alpha+1,\alpha+1}(l) v_{\alpha,\alpha}(q)(\hbar\bar{\omega} - \epsilon_\alpha + \epsilon_{\alpha+1} + \hbar\omega_q)^{-1}$$
$$- j_\alpha^* v_{\alpha,\alpha}(l) v_{\alpha+1,\alpha+1}(q)(\hbar\bar{\omega} - \epsilon_\alpha + \epsilon_{\alpha+1} - \hbar\omega_q)^{-1}$$
$$+ \sum_\mu j_\alpha^* v_{\mu,\alpha+1}(l) v_{\alpha+1,\mu}(q)(\hbar\bar{\omega} - \epsilon_\alpha + \epsilon_\mu + \hbar\omega_q)^{-1} \tag{3.21}$$

where $\delta E_\alpha \equiv (\epsilon_{\alpha+1} - \epsilon_\alpha)$ and $\delta f_\alpha \equiv (f_{\alpha+1} - f_\alpha)$; f_α is the Fermi distribution. We substitute the above elements into equations (3.12) and (3.14), and obtain the conductivity and the functions of the relaxation factors, as

$$\sigma_{zz}(\bar{\omega}) = \sum_\alpha \left(\frac{\mathrm{i}}{\bar{\omega}}\right) j_\alpha \langle Z(\bar{\omega})\rangle_\alpha + \sum_\alpha \left(\frac{\mathrm{i}}{\bar{\omega}}\right) j_\alpha^* \langle\langle Z(\bar{\omega})\rangle\rangle_\alpha \tag{3.22}$$

$$\langle Z(\omega)\rangle_\alpha = \frac{j_\alpha \delta f_\alpha}{\hbar(\bar{\omega} - \omega_\alpha) + \hbar\bar{\omega}\Gamma_P} \tag{3.23}$$

$$\langle\langle Z(\omega)\rangle\rangle_\alpha = \frac{-j_\alpha^* \delta f_\alpha}{\hbar(\bar{\omega} + \omega_\alpha) + \hbar\bar{\omega}\Gamma_Q} \tag{3.24}$$

where

$$\begin{aligned}\Gamma_P = \sum_q \frac{1}{j_\alpha} \Bigg\{ & 2\left(\frac{1}{\hbar\bar{\omega}}\right)^3 [-2 j_\alpha^* v_{\alpha,\alpha+1}(q) v_{\alpha,\alpha+1}(q)] \delta E_\alpha \\ & -2 \frac{(1/\hbar\bar{\omega})^2 \delta E_\alpha^2}{\hbar\bar{\omega} + \delta E_\alpha} \left[-\frac{j_\alpha^* v_{\alpha+1,\alpha}(q) v_{\alpha+1,\alpha}(q)}{\hbar\bar{\omega} + \hbar\omega_q} - \frac{j_\alpha^* v_{\alpha+1,\alpha}(q) v_{\alpha+1,\alpha}(q)}{\hbar\bar{\omega} - \hbar\omega_q} \right] \\ & + \left(\frac{1}{\hbar\bar{\omega}}\right) \sum_\mu \frac{j_\alpha v_{\alpha+1,\mu}(q) v_{\mu,\alpha+1}(q)}{\hbar\bar{\omega} - \epsilon_\mu + \epsilon_\alpha - \hbar\omega_q} - \left(\frac{1}{\hbar\bar{\omega}}\right) \frac{j_\alpha v_{\alpha,\alpha}(q) v_{\alpha+1,\alpha+1}(q)}{\hbar\omega - \epsilon_{\alpha+1} + \epsilon_\alpha + \hbar\omega_q} \\ & - \left(\frac{1}{\hbar\bar{\omega}}\right) \frac{j_\alpha v_{\alpha+1,\alpha+1}(q) v_{\alpha,\alpha}(q)}{\hbar\omega - \epsilon_{\alpha+1} + \epsilon_\alpha - \hbar\omega_q} + \left(\frac{1}{\hbar\bar{\omega}}\right) \sum_\mu \frac{j_\alpha v_{\mu,\alpha}(l) v_{\alpha,\mu}(q)}{\hbar\bar{\omega} - \epsilon_{\alpha+1} + \epsilon_\mu + \hbar\omega_q} \\ & - \left(\frac{1}{\hbar\bar{\omega}}\right) \frac{j_\alpha^* v_{\alpha+1,\alpha}(q) v_{\alpha+1,\alpha}(q)}{\hbar\bar{\omega} + \hbar\omega_q} - \left(\frac{1}{\hbar\bar{\omega}}\right) \frac{j_\alpha^* v_{\alpha+1,\alpha}(q) v_{\alpha+1,\alpha}(q)}{\hbar\bar{\omega} - \hbar\omega_q} \Bigg\} \tilde{N}_q\end{aligned} \tag{3.25}$$

and

$$\begin{aligned}\Gamma_Q = \sum_q \sum_\alpha \frac{1}{j_\alpha^*} \Bigg\{ & 2\left(\frac{1}{\hbar\bar{\omega}}\right)^3 [-2 j_\alpha v_{\alpha,\alpha+1}(q) v_{\alpha,\alpha+1}(q)] \delta E_\alpha \\ & +2 \frac{(1/\hbar\bar{\omega})^2 \delta E_\alpha^2}{\hbar\bar{\omega} + \delta E_\alpha} \left[-\frac{j_\alpha v_{\alpha,\alpha+1}(q) v_{\alpha,\alpha+1}(q)}{\hbar\bar{\omega} - \hbar\omega_q} - \frac{j_\alpha v_{\alpha,\alpha+1}(q) v_{\alpha,\alpha+1}(q)}{\hbar\bar{\omega} - \hbar\omega_q} \right] \\ & + \left(\frac{1}{\hbar\bar{\omega}}\right) \sum_\mu \frac{j_\alpha^* v_{\alpha,\mu}(q) v_{\mu,\alpha}(q)}{\hbar\bar{\omega} - \epsilon_\mu + \epsilon_{\alpha+1} - \hbar\omega_q} - \left(\frac{1}{\hbar\bar{\omega}}\right) \frac{j_\alpha^* v_{\alpha+1,\alpha+1}(q) v_{\alpha,\alpha}(q)}{\hbar\bar{\omega} - \epsilon_\alpha + \epsilon_{\alpha+1} + \hbar\omega_q} \\ & - \left(\frac{1}{\hbar\bar{\omega}}\right) \frac{j_\alpha^* v_{\alpha,\alpha}(q) v_{\alpha+1,\alpha+1}(q)}{\hbar\bar{\omega} - \epsilon_\alpha + \epsilon_{\alpha+1} - \hbar\omega_q} + \left(\frac{1}{\hbar\bar{\omega}}\right) \sum_\mu \frac{j_\alpha^* v_{\mu,\alpha+1}(q) v_{\alpha+1,\mu}(q)}{\hbar\bar{\omega} - \epsilon_\alpha + \epsilon_\mu + \hbar\omega_q} \\ & - \left(\frac{1}{\hbar\bar{\omega}}\right) \frac{j_\alpha v_{\alpha,\alpha+1}(q) v_{\alpha,\alpha+1}(q)}{\hbar\bar{\omega} + \hbar\omega_q} - \left(\frac{1}{\hbar\bar{\omega}}\right) \frac{j_\alpha v_{\alpha,\alpha+1}(q) v_{\alpha,\alpha+1}(q)}{\hbar\bar{\omega} - \hbar\omega_q} \Bigg\} \tilde{N}_q.\end{aligned} \tag{3.26}$$

Here Γ_P and Γ_Q are functions of the lineshape which contain the line shift and line half width in a resonant system. The first, second, seventh and eighth terms of equations (3.25) and (3.26) do not appear in other theories [6,10]. These correction terms may give rise to some interesting effects, particularly in a low-dimensional quantum system.

4. Conclusion

In this paper, we have introduced a method of deriving a conductivity formula which includes the relaxation function in a quasi-two-dimensional system. We used an operator algebra technique for the expansion of the propagators which are included in the conductivity tensor. Following the usual projection technique, we expanded the relaxation factors contained in the conductivity tensor with the help of projectors and obtained Lorentz-like formulae. A parabolic confinement potential was chosen.

We showed how to calculate the relaxation function using projectors. We expanded the propagators via a series of diagonal projectors as is commonly performed in quantum transport theories [6, 10]. The properties of the projection operators in this system are quite different from the case of three-dimensional conductivity.

Through systematically using projection operators, we have derived the useful relation of equation (3.12). Using this relation and systematically calculating the matrix elements, we obtained a conductivity formalism comprising functions of relaxation factors. These functions contain the line shift and line half width in the resonant system. The first, second, seventh and eighth terms of equations (3.25) and (3.26) do not appear in other theories [10]. These terms are expected to play an effective role in low-dimensional quantum systems. The origin of these terms is from the properties of our projection operators in this system.

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